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Stability of elastic and viscoelastic plates in a gas flow taking into account shear strains

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Abstract

It is well known that the internal friction in a material can have a considerable destabilizing effect on the stability of non-conservative systems. Apart from the Voigt model, the viscoelastic body model is sometimes utilized to describe material damping. This relates the stability problem for non-conservative elastic systems with that for viscoelastic system. The Bubnov–Galerkin method is usually applied for solving the problems. In this case, the displacement functions are represented by series in terms of natural vibration modes $\varphi_i(\mathbf{x})$ of the elastic system. To provide a high degree of accuracy for the solution, one should involve a fairly large number of modes. For a viscoelastic plate, the number of terms to be kept in the expansion of the deflection can be substantially more. One should bear in mind, however, that as the number of modes preserved in the expansion increases, the influence of shear strains and rotational inertia on the behavior of the solution becomes more pronounced. In view of this, it is important to study the stability of non-conservative viscoelastic systems with the shear strain and rotational inertia being taken into account. In the present paper this problem is solved for a viscoelastic plate in a supersonic gas flow.

1. Introduction

It is well known that the internal friction in a material can have a considerable destabilizing effect on the stability of non-conservative systems. This fact was first mentioned by Ziegler [1] and later in many other works [2–7]. Apart from the Voigt model, the viscoelastic body model is sometimes utilized to describe material damping [3,8–10]. This relates the stability problem for non-conservative elastic systems with that for viscoelastic systems [10–12]. The Bubnov–Galerkin method is usually applied for solving the problems. In this case, the displacement functions are

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represented by series in terms of natural vibration modes $\varphi_i(\mathbf{x})$ of the elastic system. For the qualitative analysis in the indicated expansion usually only the first two forms are considered. However, to provide a high degree of accuracy for the solution, one should involve a fairly large number of modes. For example, in the problem of the stability of an elastic plate in a gas flow not less than four terms must be taken into account in the expansion of the plate deflection [13,14]. For a viscoelastic plate, the number of terms to be kept in the expansion of the deflection can be substantially more [10,12]. One should bear in mind, however that as the number of modes preserved in the expansion increases, the influence of shear strains and rotational inertia on the behavior of the solution becomes more pronounced [15]. In view of this, it is important to study the stability of non-conservative viscoelastic systems with the shear strain and rotational inertia being taken into account. In the present paper this problem is solved for a viscoelastic plate in a supersonic gas flow.

2. The statement of the problem

Consider an infinitely long viscoelastic plate with freely supported longer edges exposed to a supersonic gas flow with a constant velocity v (Fig. 1). The plate is subjected to a load in the plane of the plate. The load $q(t)$, which is uniformly distributed along the movable edge, is applied in the middle plane of the plate. Restricting the consideration to the case of cylindrical bending, it is assumed that the plate deflection w is a function of the single space co-ordinate x and time t, i.e., $w = w(t, x)$.

By considering shear strains the Timoshenko hypothesis will be used. In accordance with this the angle, formed by a tangent to the middle surface of the plate with the axis x, is defined by the sum

$$\partial w/\partial x = \psi + \theta. \tag{1}$$

The rotation angle ψ is stipulated by bending of the plate and is connected with the bending moment M_x by the relation

$$M_x = -D(1 - \mathbf{R})\partial\psi/\partial x. \tag{2}$$

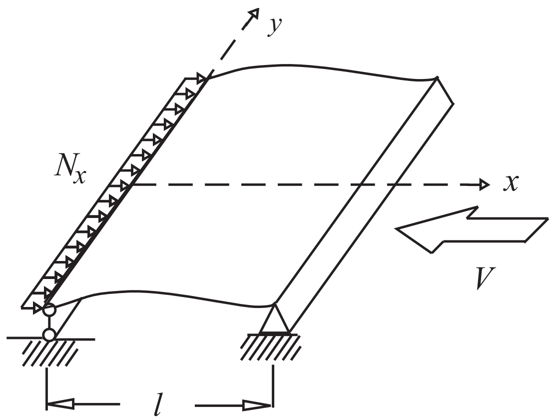

Fig. 1. The plate in a gas flow.

Here $D = E\delta^3/[12(1-\mu^2)]$ is the cylindrical stiffness of the plate, E, μ are the elasticity modulus and the Poisson ratio of the plate material, δ is the plate thickness, $\mathbf{R}$ is the relaxation operator of the material

$$\mathbf{R}\frac{\partial\psi}{\partial x} = \int_0^t R(t-\tau)\frac{\partial\psi(\tau,x)}{\partial x}\,\mathrm{d}\tau,$$

$$0 \leqslant \int_0^\infty R(\tau)\,\mathrm{d}\tau \leqslant 1, \quad R(\tau) \geqslant 0.$$

The angle θ is defined by the shear strain and is bound with the transverse force Q_x by the equality

$$Q_x = k'\delta G(1-\mathbf{B})\theta, \tag{3}$$

where G is the shear modulus, k' is the numerical coefficient, considering the transverse shear, $\mathbf{B}$ is the operator of the shear relaxation,

$$\mathbf{B}\theta = \int_0^t B(t-\tau)\theta(\tau)\,\mathrm{d}\tau,$$

$$0 \leqslant \int_0^\infty B(\tau)\,\mathrm{d}\tau \leqslant 1, \quad B(\tau) \geqslant 0.$$

Using the equilibrium equations and Eq. (1) gives the equations of motion of the plate, taking into account the rotational inertia, transverse shear strains and external damping as

$$D(1-\mathbf{R})\frac{\partial^2\psi}{\partial x^2} + k'\delta G(1-\mathbf{B})\left(\frac{\partial w}{\partial x} - \psi\right) - \rho J\frac{\partial^2\psi}{\partial t^2} = 0,$$

$$\rho\delta\frac{\partial^2 w}{\partial t^2} + k_*\rho\delta\frac{\partial w}{\partial t} - k'\delta G(1-\mathbf{B})\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial\psi}{\partial x}\right) + N_x\frac{\partial^2 w}{\partial x^2} + q = 0. \tag{4}$$

Here ρ is the density of the plate material, q is the aerodynamic load, defined by the piston theory

$$q = \frac{p_\infty V}{M}\left(V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right),$$

where M is the Mach constant and p_∞ is the pressure in the unperturbed gas flow.

The terms

$$-\rho J\,\partial^2\psi/\partial t^2, \quad -\rho\delta\,\partial^2 w/\partial t^2, \quad -k_*\rho\delta\,\partial w/\partial t$$

determine, respectively, the magnitude of the rotation inertia loads, the transverse inertia load and the damping load, and N_x is the intensity of forces, acting in the middle surface of the plate.

The solution of equations (4) must satisfy boundary conditions: at $x = 0$ and $x = l$, $w = 0$; $M_x = 0$ or $\partial\psi/\partial x = 0$ and initial conditions at $t = 0$, $w(0) = w_o(x)$, $\partial w/\partial t = v_o$, $\psi(0) = \psi_o$, $\partial\psi/\partial t = u_o(x)$.

An approximate solution of equations (4) is sought in the form

$$w(t,x) = \sum_{i=1}^n f_i(t)\sin\frac{i\pi}{l}x, \quad \psi(t,x) = \sum_{i=1}^n \varphi_i(t)\cos\frac{i\pi}{l}x, \tag{5}$$

where $f_i(t)$, $\varphi_i(t)$ are unknown functions of time.

To obtain the functions $f_i(t)$, $\varphi_i(t)$ the Bubnov–Galerkin–Kantorovich method will be used.

Substituting expressions (5) into Eq. (4), multiplying the first of them by $\cos(i\pi/l)x$, the second by $\sin(i\pi/l)x$ and integrating with respect to x on the interval $[0, l]$, gives as the final result

$$\ddot{\varphi}_i + \frac{D}{\rho J}\frac{i^2\pi^2}{l^2}(1-\mathbf{R})\varphi_i - k'\frac{\delta G}{\rho J}(1-\mathbf{B})\left(\frac{i\pi}{l}f_i - \varphi_i\right) = 0,$$

$$\ddot{f}_i + k_*\dot{f}_i + k'\frac{i\pi}{\rho l}G(1-\mathbf{B})\left(\frac{i\pi}{l}f_i - \varphi_i\right)$$

$$-\frac{i^2\pi^2}{l^2}\frac{N_x}{\rho\delta}f_i + \frac{p_\infty V}{\rho\delta M}\dot{f}_i + \frac{4p_\infty V^2}{Ml\rho\delta}\sum_{j=1}^{n} b_{ij}f_j = 0. \tag{6}$$

Here the dot denotes the derivative with respect to time t,

$$b_{ij} = \begin{cases} \frac{1}{2}\frac{ij}{i^2-j^2}[1-(-1)^{i+j}], & i \neq j, \\ 0, & i = j. \end{cases}$$

Now by introducing the dimensionless time $t_1 = \omega_1 t$, where ω_1 is the minimum frequency of proper transverse oscillations of the isotropic elastic plate,

$$\omega_1 = (\pi^4 D)/(\rho\delta l^4).$$

Then Eq. (6) can be rewritten in the dimensionless form (for further convenience the derivative with respect to t_1 is denoted again by the dot)

$$\ddot{\varphi}_i + i^2\beta(1-\mathbf{R})\varphi_i - (1-\mu^2)k'\frac{G}{E}\beta^2(1-\mathbf{B})\left(\frac{i\pi\delta}{l}c_i - \varphi_i\right) = 0,$$

$$\ddot{c}_i + 2\varepsilon\dot{c}_i + (1-\mu^2)k'\frac{G}{E}\beta\frac{il}{\pi\delta}(1-\mathbf{B})\left(\frac{i\pi\delta}{l}c_i - \varphi_i\right) - i^2\alpha c_i + v\sum_{i=1}^{n} b_{ij}c_j = 0, \tag{7}$$

where

$$c_i = f_i/\delta, \quad \beta = 12l^2/(\pi^2\delta^2), \quad \alpha = N_x l^2/(\pi^2 D),$$
$$2\varepsilon = k_*/\omega_1 + p_\infty V/(\rho\omega_1\delta M), \quad v = 4p_\infty V^2/(\rho\delta l M\omega_1^2).$$

The solution of equations (7) must satisfy to initial conditions

$$c_i(0) = \frac{2}{\delta l}\int_0^l w_o(x)\sin\frac{i\pi}{l}x\,\mathrm{d}x, \quad \varphi_i(0) = \frac{2}{l}\int_0^l \psi_o(x)\cos\frac{i\pi}{l}x\,\mathrm{d}x,$$

$$\dot{c}_i(0) = \frac{2}{\delta l}\int_0^i v_o(x)\sin\frac{i\pi}{l}x\,\mathrm{d}x, \quad \dot{\varphi}_i(0) = \frac{2}{l}\int_o^l u_o(x)\cos\frac{i\pi}{l}x\,\mathrm{d}x.$$

Neglecting rotary inertia, the first equation of Eq. (6), becomes

$$\frac{i^2\pi^2}{l^2}D(1-\mathbf{R})\varphi_i - k'\delta G(1-\mathbf{B})\left(\frac{i\pi}{l}f_i - \varphi_i\right) = 0.$$

From this equation the difference $((i\pi/l)f_i - \varphi_i)$ can be expressed as

$$\frac{i\pi}{l}f_i - \varphi_i = \left[(1-\mathbf{R}) + \frac{k'\delta G l^2}{i^2\pi^2 D}(1-\mathbf{B})\right]^{-1}(1-\mathbf{B})\frac{i\pi}{l}f_i.$$

Substituting this expression into the second equation of system (6) gives

$$(1-\mathbf{B})^{-1}\left[\ddot{f}_i + \left(k_* + \frac{p_\infty V}{\rho\delta M}\right)\dot{f}_i - \frac{i^2\pi^2}{l^2}\frac{N_x}{\rho\delta}f_i + \frac{4p_\infty V^2}{Ml\delta\rho}\sum_{j=1}^{n} b_{ij}f_i\right] + k'\frac{i^2\pi^2}{\rho l^2}G\left[(1-\mathbf{R}) + k'\frac{\delta G l^2}{i^2\pi^2 D}(1-\mathbf{B})\right]^{-1}(1-\mathbf{R})f_i = 0. \quad (8)$$

Now multiply Eq. (8) from the left with $[(1-\mathbf{R}) + k'\delta G l^2/(i^2\pi^2 D)(1-\mathbf{B})]$ and further with $(1-\mathbf{R})^{-1}$. Then

$$\left[(1-\mathbf{B})^{-1} + k'\frac{G\delta l^2}{i^2\pi^2 D}(1-\mathbf{R})^{-1}\right] \times \left[\ddot{f}_i + \left(k_* + \frac{p_\infty V}{\rho\delta M}\right)\dot{f}_i - \frac{i^2\pi^2}{l^2}\frac{N_x}{\rho\delta}f_i + \frac{4p_\infty V^2}{Ml\delta\rho}\sum_{j=1}^{n} b_{ij}f_i\right] + k'\frac{i^2\pi^2}{\rho l^2}Gf_i = 0.$$

Solving this equation with respect to $\ddot{f}_i$ gives

$$\ddot{f}_i + \left(k_* + \frac{p_\infty V}{\rho\delta M}\right)\dot{f}_i - \frac{i^2\pi^2}{l^2}\frac{N_x}{\rho\delta}f_i + \frac{4p_\infty V^2}{Ml\rho\delta}\sum_{j=1}^{n} b_{ij}f_j + \left[(1-\mathbf{B})^{-1} + k'\frac{G\delta l^2}{i^2\pi^2 D}(1-\mathbf{R})^{-1}\right]^{-1}k'\frac{i^2\pi^2}{\rho l^2}Gf_i = 0. \quad (9)$$

Using the previous dimensionless values, Eq. (9) can be rewritten in the form

$$\ddot{c}_i + 2\varepsilon\dot{c}_i - i^2\alpha c_i + v\sum_{j=1}^{n} b_{ij}c_j + [(1-\mathbf{R}^{-1}) + a(1-\mathbf{B})^{-1}]^{-1}i^4 c_i = 0, \quad (10)$$

where

$$a = (i^2/\beta)\,E/(k'(1-\mu^2)G).$$

The operators $(1-\mathbf{R})^{-1}$ $(1-\mathbf{B})^{-1}$ are operators of the tension–compression $(1+\mathbf{K})$ and shear $(1+\mathbf{U})$ creep, respectively.

Then

$$\ddot{c}_i + 2\varepsilon\dot{c}_i - i^2\alpha c_i + v\sum_{j=1}^{n} b_{ij}c_j + [(1+\mathbf{K}) + a(1+\mathbf{U})]^{-1}i^4 c_i = 0. \quad (11)$$

The integral operator $[(1+\mathbf{K}) + a(1+\mathbf{U})]^{-1}$ can be represented in the following way:

$$[(1+\mathbf{K}) + a(1+\mathbf{U})]^{-1} = \frac{1}{1+a}\left[1 + \frac{1}{1+a}(\mathbf{K} + a\mathbf{U})\right]^{-1} = \frac{1}{1+a}(1-\mathbf{R}_*),$$

moreover

$$\mathbf{R}_* = \frac{1}{1+a}(\mathbf{K}+a\mathbf{U})\left[1+\frac{1}{1+a}(\mathbf{K}+a\mathbf{U})\right]^{-1}.$$

The kernel of the operator $\mathbf{R}_*$ is the resolvent for the kernel of the operator $1/(1+a)(\mathbf{K}+a\mathbf{U})$ and it can be obtained with help of the Laplace transformation. Finally Eq. (11) can be written in the form

$$\ddot{c}_i + 2\varepsilon\dot{c}_i - i^2\alpha c_i + v\sum_{j=1}^{n} b_{ij}c_j + \frac{i^4}{1+a}(1-\mathbf{R}_*)c_i = 0. \tag{12}$$

3. The stability of the equilibrium state of the plate

For the stability of the zero solution of equations (7) or (11) one can rewrite them transfering the lower limit in the integrals to $-\infty$. In such a case the solution, for instance, of Eqs. (7) can be sought as the products

$$\varphi_i(t) = a_i e^{i_*\omega t}, \quad c_i(t) = b_i e^{i_*\omega t}. \tag{13}$$

Here a_i, b_i are constants, ω is the frequency and $i_* = \sqrt{-1}$.

In this connection the integral term $\mathbf{R}\varphi_i$ assumes the form

$$\begin{aligned}\int_{-\infty}^{t} R(t-\tau)\varphi_i(\tau)\,d\tau &= \int_{-\infty}^{t} R(t-\tau)e^{i_*\omega\tau}\,d\tau a_i \\ &= \int_0^{\infty} R(\tau_1)e^{i_*\omega(t-\tau_1)}\,d\tau_1 a_i = e^{i_*\omega t}\int_0^{\infty} R(\tau_1)e^{-i_*\omega\tau_1}\,d\tau_1 a_i.\end{aligned}$$

A similar transformation can be fulfilled with the term $\mathbf{B}(i\pi\delta c_i/l - \varphi_i)$. After the substitution of expressions (13) into Eqs. (7) one obtains the system of linear homogeneous algebraic equations

$$\begin{aligned}&-\omega^2 a_i + i^2\beta(1-R^*)a_i - (1-\mu^2)k'\frac{G}{E}\beta^2(1-B^*)\left(\frac{i\pi}{l}b_i - a_i\right) = 0,\\ &-\omega^2 b_i + 2\varepsilon i_*\omega b_i + (1-\mu^2)k'^2\frac{G}{E}\beta\frac{il}{\pi\delta}(1-B^*)\left(\frac{i\pi\delta}{l}b_i - a_i\right)\\ &\quad - i^2\alpha b_i + v\sum_{j=1}^{n} b_{ij}b_j = 0,\end{aligned} \tag{14}$$

where

$$R^* = \int_0^{\infty} R(\tau_1)e^{-i_*\omega\tau_1}\,d\tau_1, \quad B^* = \int_0^{\infty} B(\tau_1)e^{-i_*\omega\tau_1}\,d\tau_1.$$

The value of ω is found as a root of the equality

$$|\mathbf{A}| = 0.$$

Here $|\mathbf{A}|$ is the determinant of the matrix of coefficients of equations (14).

The critical value, for example, of the parameter α is found from the condition that the real part of ω is equal to zero. However, the finding of complex-valued roots of the transcendental equation is not an easy problem. Therefore for the investigation of the stability of the momentless equilibrium state of the plate the method of the top Lyapunov exponent is applied, which is defined by the expression

$$\lambda = \lim_{t \to \infty} (\|\mathbf{Y}(t)\|/\|\mathbf{Y}(0)\|),$$

where $\|\mathbf{Y}(t)\|$, $\|\mathbf{Y}(0)\|$ are norms of solutions of equations (7) in Euclidean space for instants t and $t = 0$.

If $\lambda < 0$, then the equilibrium state of the plate is asymptotically stable in Lyapunov sense, and vice versa, if $\lambda > 0$, then the equilibrium state is unstable. For the calculation of the top Lyapunov exponent in that case, if Eqs. (7) are differential (at $\mathbf{R} = 0$, $\mathbf{B} = 0$) or are reduced to them (as, for example, in a case of degenerate kernels of integral operators $\mathbf{R}$, $\mathbf{B}$), one can use the numerical method, proposed in Ref. [16]. If integro-differential equations (7) cannot be reduced to differential equations, then for the calculation of λ, the method described in Ref. [17], can be applied.

4. Example

By way of an example consider a plate, the relaxation kernels of the material of which are $R(t-\tau)$, $B(t-\tau)$ having forms

$$R(t-\tau) = \chi L \mathrm{e}^{-\chi(t-\tau)}, \quad B(t-\tau) = \eta H \mathrm{e}^{-\eta(y-\tau)}, \tag{15}$$

where R, B are measures of the axial and the shear relaxation, respectively, $0 \leqslant L \leqslant 1$, $0 \leqslant H \leqslant 1$, χ, η are parameters, characterizing the time of the relaxation.

For similar relaxation kernels integro-differential equations (7) can be reduced to the system of differential equations. With this purpose one can use new unknowns

$$z_i = \mathbf{R}\varphi_i, \quad x_i = \mathbf{B}((i\pi\delta/l)c_i - \varphi_i).$$

These integral relations are equivalent to differential equalities

$$\dot{z}_i = \chi(L\varphi_i - z_i), \quad \dot{x}_i = \eta[H((i\pi\delta/l)c_i - \varphi_i) - x_i]$$

with initial conditions $z_i(0) = 0$, $x_i(0) = 0$.

Further the Poisson coefficient μ and the dimensionless value $(1-\mu^2)k'G/E$ are assumed, respectively, equal to 0.15 and 0.2, and the ratio $(1-\mu^2)l/(\pi\delta)$ to 5. Critical values of the parameter α_{cr} obtained with help of Eqs. (7) for the elastic plate depending on the parameter v and on the number of terms in the expansion of the plate deflection are presented in Table 1. The parameter ε is equal to 0.1.

In the same table values of the parameter α_*, found without considering transverse shear strains and the rotation inertia, are given.

For the estimation of the effect of shear strains only in the same table quantities of the parameter α_{sh}, which are received from Eqs. (12), are written.

The relation between the parameters α_{sh} and v is plotted in Fig. 2 as the solid line. To produce this curve additional results for interval $v \in (4; 6)$ were obtained with the step $\Delta v = 0.25$. The line is

Table 1
Magnitudes of the critical parameter α for the elastic plate for varying v and the number of terms n in the expansion of the deflection at $\varepsilon = 0.1$

v	α_*			α_{cr}			α_{sh}		
	$n=2$	$n=4$	$n=6$	$n=2$	$n=4$	$n=6$	$n=2$	$n=4$	$n=6$
0	1.00	1.00	1.00	0.98	0.98	0.98	0.98	0.98	0.98
1	1.04	1.04	1.04	1.02	1.02	1.02	1.02	1.02	1.02
2	1.16	1.16	1.16	1.15	1.15	1.15	1.16	1.15	1.16
3	1.38	1.38	1.38	1.41	1.40	1.40	1.41	1.40	1.40
4	1.81	1.77	1.77	2.00	1.91	1.91	2.00	1.90	1.90
5	2.78	3.00	3.01	2.45	2.71	2.71	2.46	2.71	2.72
6	2.34	2.64	2.65	2.01	2.35	2.36	2.02	2.36	2.37
10	0.57	1.28	1.29	0.24	1.03	1.05	0.24	1.05	1.07

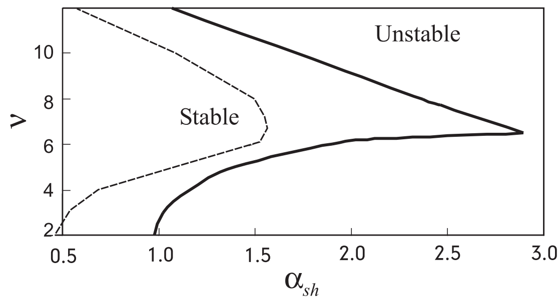

Fig. 2. Critical quantity α_{sh} versus parameter v for elastic and viscoelastic plates.

obtained for the parameter, characterizing the external and aerodynamical damping, ε equal to 0.1. It is interesting, that at $\varepsilon = 0.05$ the similar curve coincides with the same solid line. The plotted curve is the boundary between regions of stable and unstable states of the plate. The form of the dependence $\alpha_* \sim v$ is well known from many works [2,12,14]. One can see that taking into account shear strains does not lead to principal changes in this form.

The comparison of values α_{cr} and α_{sh} shows that at chosen input data the rotation inertia does not render practically any influence on the magnitude of the critical parameter α. From another side it is obvious that taking into account shear strains leads to corrections in the magnitude of the critical parameter, the value of which increases with the increase of the parameter v. If at $v = 0$ the difference between related values α_{cr} and α_* achieves only 2%, then at $v = 6$ it is already more than 10%.

Simultaneously presented results demonstrate, that for obtaining sufficiently exact results at assumed input data in the expansion of the deflection, not less than 4 terms must be kept. This fact is known too from previous works, where the stability of the isotropic plate without taking into account shear strains was considered. The effect of shear strains and rotational inertia increases with increasing numbers of sine half-waves in the expansion of the plate deflection. This circumstance was discussed for bars by Timoshenko [15].

Table 2
Magnitudes of the critical parameter α for the viscoelastic plate for varying n in the expansion of the deflection at $\varepsilon = 0.1$; $L = 1.0$; $\chi = 0.01$; $H = 1.0$; $\eta = 0.01$; $v = 5.0$

n	α_{cr}	α_*
2	2.40	2.65
4	0.98	0.18
6	1.03	0.19
8	1.04	0.19

Calculations for other values of input data show, that the minimum necessary number of terms n can be more than four, where it can be different depending on that fact whether shear strains are taking into account or not.

For the confirmation of it in Table 2 quantities of parameters α_{cr} and α_*, found for the viscoelastic plate with the maximum relaxation measure at the different number of terms in the expansion of the deflection, are presented. From here it can be seen, that in the isotropic plate it is enough to consider only the first four terms in the expansion of the deflection. However, if one considers shear strains too, then not less as 6–8 terms should be taking into account. The difference in quantities of the critical parameter α, found at $n = 2$ and 8 proves to be more essential than for the elastic plate. Simultaneously these data show, that by considering shear strains and rotational inertia leads to the sharp variation of values of the parameter α_{cr} in the comparison with the value α_*, found at the same number of terms in the expansion of the plate deflection.

For the comparison of relations $v \sim \alpha_{cr}$ for elastic and viscoelastic plates in Fig. 2 the dash line is built, received at $\varepsilon = 0.05$ for the viscoelastic plate with parameters $L = H = 0.5$ and $\chi = \eta = 0.1$ and $n = 4$. These curves show that taking into account the viscous properties of the material can induce an appreciable change in the magnitude of the critical parameter α_{cr} and even in the form of the curve $v \sim \alpha_{cr}$.

Data, presented in Tables 3 and 4 make possible to estimate the influence of viscoelastic characteristics of the material and of the parameter ε on the magnitude of the critical parameter α. From the comparison of results, containing in Table 3, it follows, that at the increase of relaxation measures of the material L, H in indicated limits the decrease of parameters α_{cr} and α_*, is observed, the speed of which increases with the increase of values L, H.

Results, presented in Table 4, show, that the change of the coefficient ε and of the parameter χ, defining the relaxation time of the material, can render essential effect on the value of critical parameter α_{cr}. It is especially appreciable in that case, if we consider shear strains and the rotation inertia.

For the estimation of the effect of shear strains only on critical values of the parameter α for the viscoelastic plate let us use Eq. (12). Following creep kernels

$$K(t-\tau) = \gamma K \mathrm{e}^{-\gamma(t-\tau)} \quad \text{and} \quad U(t-\tau) = \xi P \mathrm{e}^{-\xi(t-\tau)}$$

correspond to relaxation kernels $R(t-\tau)$ and $B(t-\tau)$ (15).

Table 3
The dependence of values of the critical parameter α for characteristics of the viscoelasticity at $\varepsilon = 0.1$; $n = 4$ and $v = 5$

L	χ	H	η	α_{cr}	α_*	α_{sh}
0.25	0.01	0.00	0.00	2.71	2.98	2.71
0.25	0.01	0.25	0.01	2.71	2.98	2.71
0.00	0.00	0.25	0.01	2.71	3.01	2.71
0.5	0.01	0.0	0.00	2.70	2.97	2.70
0.5	0.01	0.5	0.01	2.70	2.97	2.70
0.0	0.00	0.5	0.01	2.71	3.01	2.72
0.75	0.01	0.00	0.00	2.68	2.73	2.68
0.75	0.01	0.75	0.01	2.67	2.73	2.68
0.00	0.00	0.75	0.01	2.71	3.01	2.71

Table 4
The dependence of the critical parameter α for the elastic and viscoelastic plate for magnitudes of parameters ε and χ at $n = 4$ and $v = 5$

ε	L	χ	H	η	α_{cr}	α_*
0.1	0.5	0.1	0.00	0.00	1.93	2.97
	0.5	0.01			2.70	2.97
	0.5	0.001			2.71	3.00
	0.0	0.00			2.71	3.01
0.05	0.5	0.01	0.0	0.00	2.58	2.64
	0.5	0.01	0.5	0.01	1.58	—
	0.0	0.00	0.5	0.01	2.67	—

Here

$$K = \frac{L}{1-L}, \quad \gamma = \frac{\chi}{1+K}, \quad P = \frac{H}{1+H}, \quad \xi = \frac{\eta}{1+P}.$$

Applying the Laplace transformation the kernel of the operator **R** is obtained in the form

$$R(t-\tau) = \frac{d_1}{1+a}\exp[-p_1(t-\tau)] + \frac{d_2}{1+a}\exp[-p_2(t-\tau)],$$

where

$$p_1 = (-b - \sqrt{b^2 - 4c})/2, \quad p_2 = (-b + \sqrt{b^2 - 4c})/2,$$

$$b = \gamma + \xi + (\gamma K + a\xi P)/(1+a), \quad c = \gamma\xi + \gamma\xi(K + aP)/(1+a),$$

$$d_1 = (A_1 p_1 + A_2)/(p_1 - p_2), \quad d_2 = -(A_1 p_2 + A_2)/(p_1 - p_2),$$

$$A_1 = \gamma K + a\xi P, \quad A_2 = \gamma\xi(K + aP).$$

From the solution of the integro-differential equation (12) the magnitude of values α_{sh} were obtained, which are presented in the Table 3.

Comparison of α_{cr} and α_{sh} shows that the rotational inertia virtual does not influence the stability of the viscoelastic plate, as was previously the case for an elastic plate, at least for the considered input data.

5. Conclusion

In the work on the example of the plate, moving in the gas flow, the effect of shear strains and the rotational inertia on the values of critical parameters in elastic and viscoelastic non-conservative systems has been investigated. It has been shown, when considering the indicated factors, particularly shear strains, they can have an essential influence on the stability of similar systems, moreover this influence is especially appreciable in viscoelastic systems. The difference in values of critical parameters, found with and without taking into account shear strains, can produce errors of not several percents but several decades percents.

For the investigation of the stability of the zero solution of integro-differential equations, with help of which the behavior of the viscoelastic plate has been described, the method of the top Lyapunov exponent is used. For its calculation the procedure, suggested in Ref. [16] for a system of differential equations and generalized in Ref. [17] for the case of integro-differential equations, is applied. Considered examples show the high efficiency of the proposed method.

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